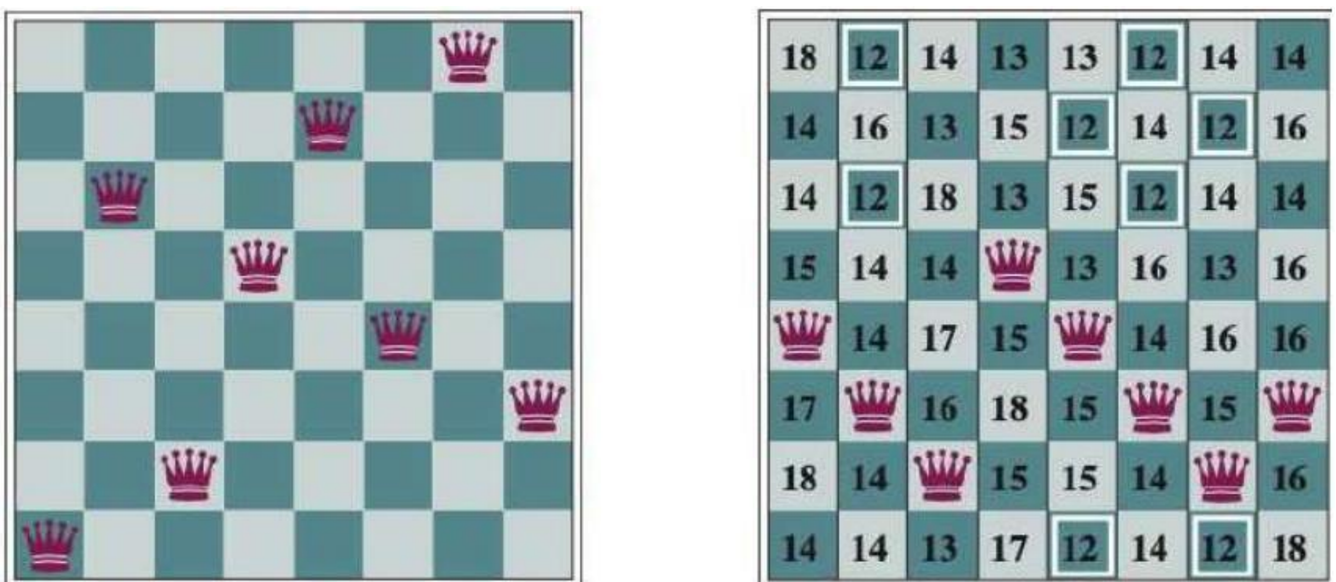
ENEM 688 – Independent Study
Lecture A: Genetic Algorithm and Lagrange Optimization

Liang Zhang
University of Maryland Eastern Shore
lzhang1@umes.edu

October 29, 2025

The 8-Queens Problem

- Every state has 8 queens, on the board, one per column.
- A queen attacks any piece in the same row, column, or diagonal.
- The initial state is chosen at random, and the successors of a state are all possible states generated by moving a single queen to another square in the same column (each state has 8×7=56 successors).
- The heuristic cost function h is the number of pairs of queens that are attacking each other; this will be zero only for solutions.



(a) (b)

Figure 4.3 (a) The 8-queens problem: place 8 queens on a chess board so that no queen attacks another. (b) An 8-queens state with heuristic cost estimate h=17.

Evolutionary Algorithms

- Evolutionary algorithms can be seen as variants of stochastic beam search that are motivated by the natural selection in biology:
- There is a population of individuals (states), in which the fittest (highest value) individuals produce offspring (successor states) that populate the next generation, a process called **recombination**.

- In genetic algorithms, each individual is a string over a finite alphabet (often a Boolean string), just as DNA is a string over the alphabet ACGT. In evolution strategies, an individual is a sequence of real numbers, and in genetic programming an individual is a computer program.

- The genetic algorithm includes:

  Initial Population, Fitness Function, Selection, Crossover, Mutation

Recall the 8-Queens Problem

- Place 8 queens on a chessboard so that no two queens attack each other.

- Constraints:
  - No two queens share the same row.
  - No two queens share the same column.
  - No two queens share the same diagonal.

Genetic Algorithm Approach

- Represent solution as a chromosome:
  - Example: 24748552 → each digit = row position of queen in that column.

- Initial population = set of candidate chromosomes.
- Operators: Selection → pick parents based on fitness.
- Crossover → combine parents to create offspring.
- Mutation → introduce random changes for diversity.

- OpenAI genetic algorithm

- More information: https://www.youtube.com/watch?v=-kpcAa-qKwY

Genetic Algorithm Approach

- Fitness function definition:

  Total pairs of queens = C(8,2) = 28.

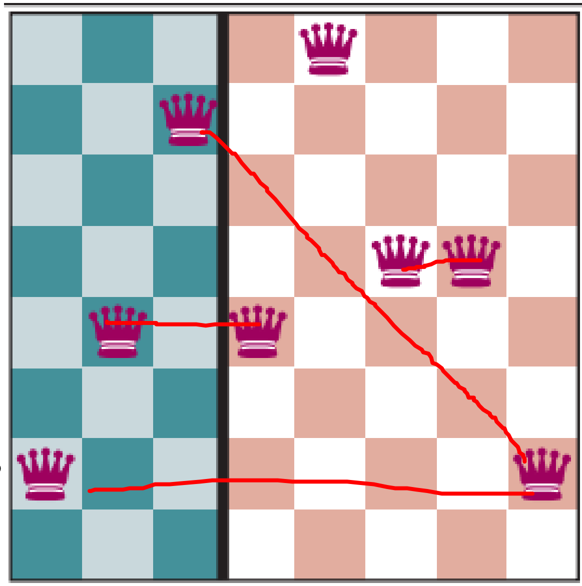
  Fitness = 28 – (number of attacking pairs)

- Why is maximum fitness = 28?
  - In the 8-queens problem, we place **8 queens** on the board. Any two queens form a pair. The number of ways to choose 2 queens from 8 is a binomial coefficient:

$$\binom{8}{2} = \frac{8 \times 7}{2} = 28$$

- **What does fitness mean?**
  - Fitness = number of **non-attacking pairs** of queens.
  - If queens never attack each other, **all 28 pairs are safe**.

- 28 = perfect solution.
- <28 = partial solution, with conflicts.



The gene is 24748552.
The fitness of the top is:
28-4=24

The Genetic Algorithm

- The selection process for selecting the individuals who will become the parents of the next generation.
- The recombination procedure.
- The mutation rate, which determines how often offspring have random mutations to their representation.
- The makeup of the next generation.
- Figure 4.6(a) shows a population of four 8-digit strings, each representing a state of the 8-queens puzzle: the c-th digit represents the row number of the queen in column c.

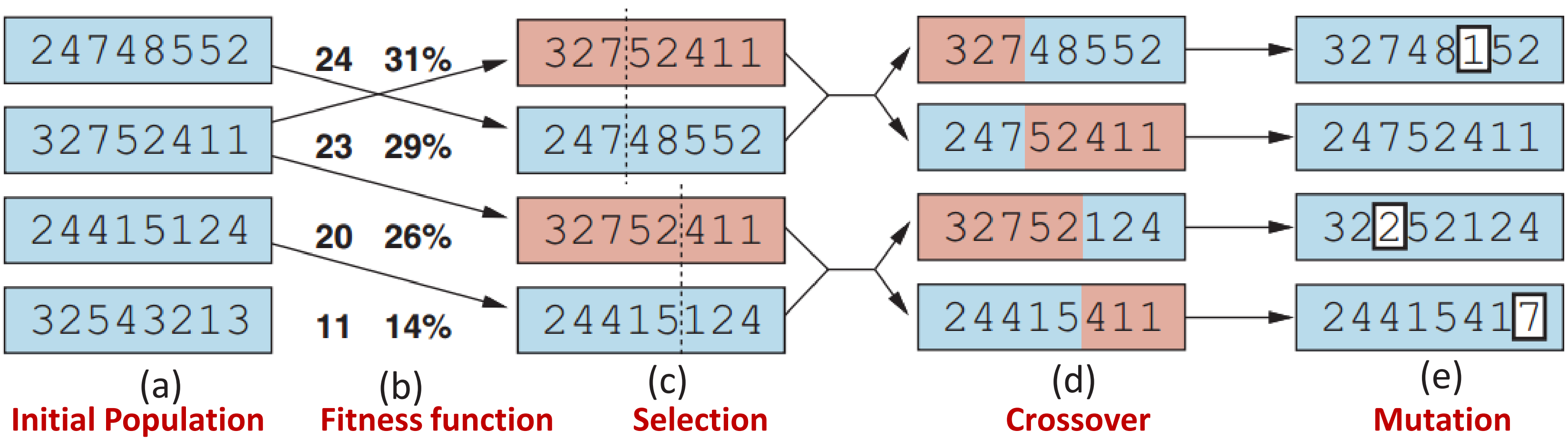
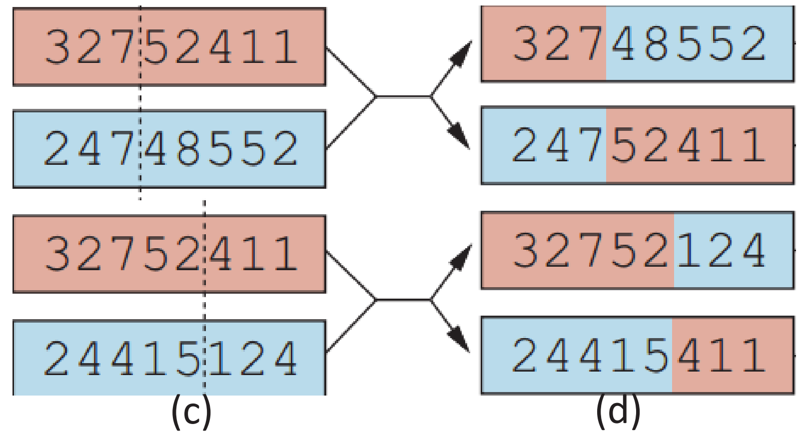


Figure 4.6 A genetic algorithm, illustrated for digit strings representing 8-queens states. The initial population in (a) is ranked by a fitness function in (b) resulting in pairs for mating in (c). They produce offspring in (d), which are subject to mutation in (e).

The Genetic Algorithm (Cont'd)

- Each representing a state of the 8-queens puzzle: the c-th digit represents the row number of the queen in column c.



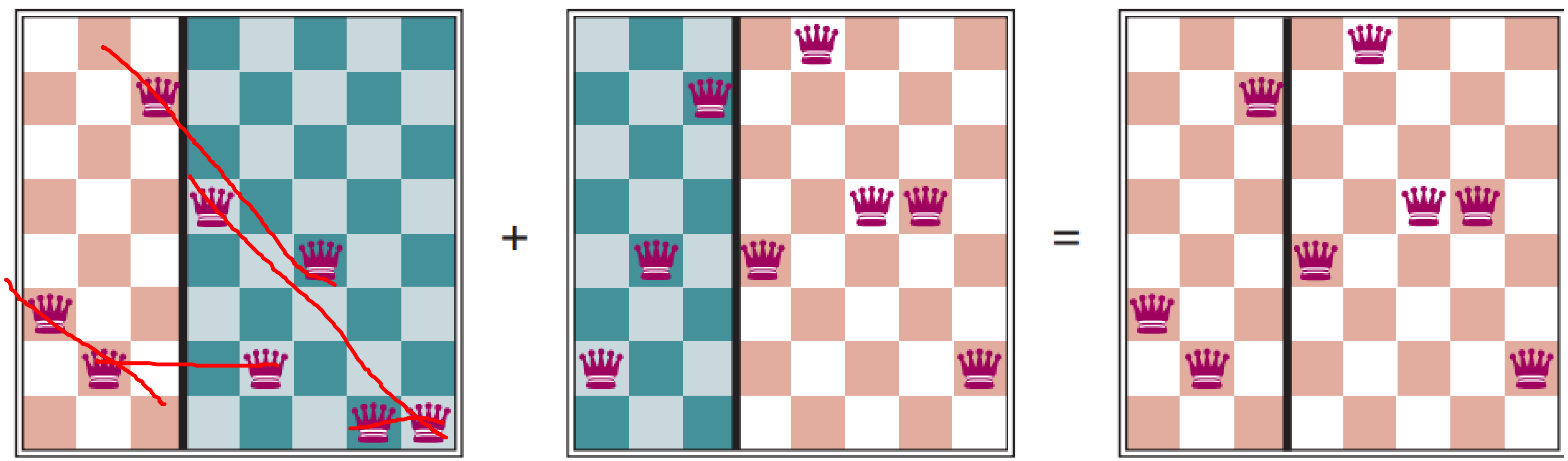


Figure 4.7 The 8-queens states corresponding to the first two parents in Figure 4.6(c) and the first offspring in Figure 4.6(d). The green columns are lost in the crossover step and the red columns are retained. (To interpret the numbers in Figure 4.6: row 1 is the bottom row, and 8 is the top row.)

The Genetic Algorithm (Cont'd)

```
function GENETIC-ALGORITHM(population, fitness) returns an individual
  repeat
     weights ← WEIGHTED-BY(population, fitness)
     population2 ← empty list
     for i = 1 to SIZE(population) do
        parent1, parent2 ← WEIGHTED-RANDOM-CHOICES(population, weights, 2)
        child ← REPRODUCE(parent1, parent2)
        if (small random probability) then child ← MUTATE(child)
        add child to population2
     population ← population2
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to fitness

function REPRODUCE(parent1, parent2) returns an individual
  n ← LENGTH(parent1)
  c ← random number from 1 to n
  return APPEND(SUBSTRING(parent1, 1, c), SUBSTRING(parent2, c + 1, n))
```

Figure 4.8 A genetic algorithm. Within the function, population is an ordered list of individuals, weights is a list of corresponding fitness values for each individual, and fitness is a function to compute these values.

Combination and Permutation

- **Permutation (Order Matters)**
- **Meaning**:
  - How many ways you can arrange things in order.
  - Different order = different arrangement.
- **Example**:
  - Seating 3 students in a row out of 5: P(5,3)=60

- **Combination (Order Doesn't Matter)**
- **Meaning**:
  - How many ways you can choose things, the order is not important.
  - Different order = same group.
- **Example**:
  - Choosing 3 markers out of 5:

$$\binom{5}{3} = \frac{5 \times 4}{2 \times 1} = 10$$

Summary of Counting Results

- Permutation of n objects:

$$n!$$

- k-permutations of n objects:

$$n!/(n-k)!$$

- Combinations of k out of n objects:

$$\binom{n}{k} = \frac{n!}{n!\,(n-k)!}$$

An Example of Continuous Variable Optimization

- We want to design a rectangular garden with a **fixed perimeter of 40 meters** and maximize the **area** A.
- Let x be the length and y be the width.
- Perimeter condition:

$$2x + 2y = 40$$

- Objective:

$$f(x) = xy$$

- Transferred Problem:

$$\min_{x,y \in \mathbb{R}} \mathrm{L}(x, y, \lambda)$$

subject to:

$$\mathrm{L}(x, y, \lambda) = -xy + \lambda \cdot (x + y - 20)$$
$$\lambda \geq 0$$

An Example of Continuous Variable Optimization

- Lagrange optimization:

$$L(x, y, \lambda) = -xy + \lambda \cdot (x + y - 20)$$

$$\frac{\partial L}{\partial x} = -y + \lambda = 0$$
$$\frac{\partial L}{\partial y} = -x + \lambda = 0$$
$$\frac{\partial L}{\partial \lambda} = x + y - 20 = 0$$

- $x = y = 10$ and $\lambda = 10$
- This is the only critical point of L

- Optimal rectangle: 10 m × 10 m (square)
- Maximum area:

$$A = 100 \ m^2$$